

Monopole, quadrupole and pairing: a shell model view

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The three main contributions to the nuclear Hamiltonian—monopole, quadrupole and pairing—are analyzed in a shell model context. The first has to be treated phenomenologically, while the other two can be reliably extracted from the realistic interactions. Due to simple scaling properties, the realistic quadrupole and pairing interactions eliminate the tendency to collapse of their conventional counterparts, while retaining their basic simplicity.

Nuclei are systems of interacting particles. Everybody agrees with this statement. But then: which interaction, which particles? These questions raise some difficulties, so the first statement is replaced by : Nuclei are systems of quasi-particles, interacting via effective Hamiltonians (or Lagrangians). The questions recur, and now the answers vary widely. Restricting attention to theorists who are content with the idea that the particles are basically neutrons and protons, there main possibilities concerning the interaction:

R. The realistic approach consists in extracting it from NN data, and then take seriously the idea that the many-body Schrödinger has to be solved exactly.

P. The phenomenological approach(es) derive the interaction from the nuclear data they are supposed to explain. There are two variants:

MFP Mean field phenomenology restricts to a maximum the number of parameters, and directs attention to global properties and general spectroscopic trends.

SMP Shell model phenomenology is prepared to introduce as many parameters as necessary to explain spectroscopic detail.

Conceptually, **R** is the most satisfactory but, in addition to its difficulty, it has suffered so far from a major drawback: The realistic interactions do not saturate well, i. e., they do not give the right binding at the right radius. Furthermore, they fail to produce the correct shell structure: doubly magic closures are missed. Much progress has been made recently in fitting the NN data perfectly, and in calculating exactly for very light nuclei. However, the problems remain, and phenomenological three-body forces have to be called in to solve them.

Historically, **P**, has enormous importance, and both its variants rest on pioneering work that remains of permanent value. It can be summarized by saying that, whatever their guise, the effective Hamiltonians must contain a monopole term that produces the spherical closures, a quadrupole one that induces deformation, and a pairing force that favours condensation at the vicinity of the Fermi surface.

My purpose is to show how these things are possible, *within a framework that starts from R*. The idea is that the trouble in realistic interactions is concentrated in the monopole part, which can be separated rigorously from the rest. Moreover, it is formally very simple and can be extracted from the data with a minimum of parameters. Once this is done, the rest of the Hamiltonian (which we call multipole), turns out to be independent of the type of realistic interaction used. What's more, it does a magnificent job in large scale shell model calculations.

The aim of these notes [1] is to provide a unified introduction to references [2,3], dealing with the monopole and multipole Hamiltonians respectively.

NOTATIONS A few equations have to exhibit explicitly angular momentum (J), and isospin (T) conservation. I will use Bruce French's product notation [4])

Γ stands for JT . Then $(-)^{\Gamma} = (-)^{J+T}$, $[\Gamma] = (2J+1)(2T+1)$, and in general $F(\Gamma) = F(J)F(T)$. Orbits are called r , s , etc., and I use $(-)^r = (-)^{j_r+1/2}$, $[r] = 2(2j_r+1)$. Expressions carry to neutron-proton formalism simply by dropping the isospin factor

m_r is the number of particles in orbit r , T_r is used for both the isospin and the isospin operator. In neutron-proton (np) scheme, m_{rx} specifies the fluid x .

$Z_{rs\Gamma}^{\dagger}$ is an operator of type $a_r^{\dagger}a_s^{\dagger}$ coupled to good spin and isospin JT . S_{rs}^{γ} is an operator of type $a_r^{\dagger}a_s$ coupled to good spin and isospin $\lambda\tau$.

V_{rstu}^{Γ} is a two body matrix element. W_{rstu}^{Γ} is used after the monopole part has been subtracted.

p is the principal oscillator quantum number.

I. THE MONOPOLE HAMILTONIAN

The only assumption that will be made is the existence of an effective potential smooth enough to do Hartee Fock (HF) variation, and capable of yielding good results.

Now: given a Hamiltonian \mathcal{H} , (\mathcal{K} is the kinetic energy)

$$\mathcal{H} = \mathcal{K} + \sum_{r \leq s, t \leq u, \Gamma} V_{rstu}^{\Gamma} Z_{rs\Gamma}^{\dagger} \cdot Z_{tu\Gamma}, \quad (1)$$

it is always possible to extract from it a monopole part \mathcal{H}_m^d , whose expectation value for any state is the average

energy of the configuration to which it belongs (a configuration is a set of states with fixed m_{rx} for each orbit). In particular, \mathcal{H}_m^d reproduces the exact energy of closed shells (cs) and single particle (or hole) states built on them $((cs) \pm 1)$, since for this set $(cs \pm 1)$ each configuration contains a single member. The result is standard [5], and we simply write it, (d stands for diagonal)

$$\mathcal{H}_m^d = \mathcal{K}^d + \sum_{rx, sx'} V_{rs}^{xx'} m_{rx} (m_{sx'} - \delta_{rs} \delta_{xx'}), \quad (2)$$

which reproduces the average energies of configurations at fixed $m_{rx} m_{rx'}$. and isospin (mT)

$$\begin{aligned} \mathcal{H}_{mT}^d = \mathcal{K}^d + \sum_{r \leq s} \frac{1}{(1 + \delta_{rs})} [a_{rs} m_r (m_s - \delta_{rs}) + \\ + b_{rs} (T_r \cdot T_s - \frac{3}{4} m \delta_{rs})], \end{aligned} \quad (3)$$

which reproduces the average energies of configurations at fixed $m_r T_r$.

Using $D_r = 2j_r + 1$, we rewrite the relevant centroids incorporating explicitly the Pauli restrictions

$$V_{rx, sx'} = \frac{\sum_J V_{rsrs}^{Jxx'} (2J+1) (1 - (-)^J \delta_{rs} \delta_{xx'})}{D_r (D_s - \delta_{rs} \delta_{xx'})} \quad (4)$$

$$V_{rs}^T = \frac{\sum_J V_{rsrs}^{JT} (2J+1) (1 - (-)^{J+T} \delta_{rs})}{D_r (D_s + \delta_{rs} (-)^T)} \quad (5)$$

$$a_{rs} = \frac{1}{4} (3V_{rs}^1 + V_{rs}^0), \quad b_{rs} = V_{rs}^1 - V_{rs}^0. \quad (6)$$

In the np scheme each orbit r goes into two rx and rx' and the centroids can be obtained through ($x \neq x'$)

$$\begin{aligned} V_{rx, sx'} = \frac{1}{2} \left[V_{rs}^1 \left(1 - \frac{\delta_{rs}}{D_r} \right) + V_{rs}^0 \left(1 + \frac{\delta_{rs}}{D_r} \right) \right] \\ V_{rx, sx} = V_{rs}^1. \end{aligned} \quad (7)$$

Under Hartree Fock variation, the m_r and T_r operators will go into non diagonal ones of the type $S_{st}^{0\tau}$. Therefore \mathcal{H}_m^d and \mathcal{H}_{mT}^d should be generalized to the full monopole Hamiltonian, \mathcal{H}_m , containing all two body quadratic forms in the $S_{st}^{0\tau}$ operators, see [6,7]). The task is not trivial, and we can avoid doing the variation explicitly, as we see next.

A. Scaling

HF variation is necessary to ensure that the system assume its correct radius, at an energy close to the correct one. In all representations \mathcal{H}_m^d has the same form, that must change smoothly as N and Z change. To discover how the evolution takes place, we first note that matrix elements for a potential of short—but non zero—range scale as the oscillator frequency $\hbar\omega$ (a consequence of the Talmi Moshinsky transformation). Then we can write

$$V(\omega)_{klmn} \cong \frac{\omega}{\omega_0} V(\omega_0)_{klmn}, \quad \hbar\omega = 34.6 A^{1/3} / \langle r^2 \rangle, \quad (8)$$

where the second equality is adapted from [8]. A very accurate fit to the spherical radii (a variant of those in [9]) yields $\langle r^2 \rangle = 0.89 \rho A^{1/3}$ with

$$\rho = A^{1/3} (1 - (2T/A)^2) e^{(3.5/A)}, \quad \text{so} \quad \hbar\omega \approx 39/\rho. \quad (9)$$

This is very simple, but it cannot be the full story, because the competition between potential and kinetic energy (which has the same scaling, $\mathcal{K}^d \equiv K = \hbar\omega/2 \sum_p (p+3/2) m_p$, m_p is the number of particles in harmonic oscillator (HO) shell p) would lead to a trivial equilibrium at $\hbar\omega = \infty$ or 0. The terms that cannot scale strictly with ω must be those that go as the total number of particles A , since they are alone responsible for saturation. We shall not try to discover the saturation mechanism, but simply note that \mathcal{H}_m^d should produce (the right) smooth contributions in A and $A^{2/3}$, and a symmetry energy in $T(T+1)/A$ and $T(T+1)/A^{4/3}$. Next we separate these “liquid drop” terms, from those that produce shell effects $\mathcal{H}_m^d = \mathcal{H}_m^{LD} + \mathcal{H}_m^s$, and set out to parametrize efficiently \mathcal{H}_m^s .

1. \mathcal{H}_m^s and the collective monopole Hamiltonian W

By *definition*, shell effects go as $A^{1/3}$, and we have assumed they can be separated those in A and $A^{2/3}$. Strutinsky’s famous theorem tells us this is possible. Here I will show an explicit example, which is the starting point in the construction of \mathcal{H}_m^s . Independently of the detailed saturation mechanism, there must be “something” in \mathcal{H}_m^d capable of cancelling K . As it is always possible to separate from \mathcal{H}_m^d a leading term in $m(m-1)/2 \equiv A(A-1)/2$, the “something” must be related to it. As the end result must go like A , this term must be scaled by A^{-1} . However, there is no such thing in \mathcal{H}_m^d if we take the scaling to be $\hbar\omega$, i. e., $A^{-1/3}$. By using the techniques of section II, we discover that the term we are after is

$$W = \sum_p (m_p / \sqrt{D_p})^2, \quad D_p = (p+1)(p+2), \quad (10)$$

which has the remarkable property that $\hbar\omega(W - 4K)/4$ cancels exactly to order $A^{1/3}$ and (as we shall see) produces strong shell effects. W is called the collective monopole Hamiltonian for reasons that will become fully apparent in section II A.

$W - 4K$ is taken to be the first contribution to \mathcal{H}_m^d , which has been fitted on the set of known $(cs) \pm 1$ spectra. For the explicit construction of the rest I refer to [2]. For the present purpose it will be sufficient to mention that it includes an $l \cdot s + l \cdot l$ piece that behaves as one-body and

has the same coefficient as $W - 4K$. Then, there are two-body terms that ensure the correct evolution of $(cs) \pm 1$ spacings from HO to EI closures (extruder-intruder, such as $NZ = 14, 28, 50$, etc). Fig 1 illustrates the shell formation mechanism for $T = 5$ nuclei. Note the beautiful HO closures generated by $W - 4K$, erased by $l \cdot s + l \cdot l$ (as in ^{30}Ne at $N = 20$), and replaced by EI closures through the two body terms.

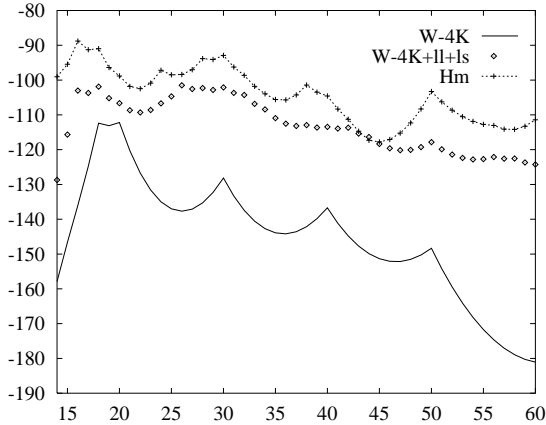


FIG. 1. Shell formation mechanism in $T = 5$ nuclei. Negative binding energies in y -axis are *repulsive*. N in x -axis

II. THE MULTIPOLE HAMILTONIAN

The multipole Hamiltonian is defined as $\mathcal{H}_M = \mathcal{H} - \mathcal{H}_m$. As we are no longer interested in the full \mathcal{H} , but its restriction to a finite space, \mathcal{H}_M will be more modestly called H_M , with monopole-free matrix elements given by

$$W_{rstu}^{JT} = V_{rstu}^{JT} - \delta_{rs}\delta_{tu}V_{rs}^T. \quad (11)$$

To concentrate on the physics I shall only write the formulae that are strictly necessary to show how the pairing plus quadrupole interactions appear in their *normalized* form of. Abundant detail is given in [3].

There are two standard ways of writing H_M :

$$H_M = \sum_{r \leq s, t \leq u, \Gamma} W_{rstu}^\Gamma Z_{rs\Gamma}^+ \cdot Z_{tu\Gamma}, \quad \text{or} \quad (12)$$

$$H_M = \sum_{rstu\Gamma} [\gamma]^{1/2} \omega_{rstu}^\gamma (S_{rt}^\gamma S_{su}^\gamma)^0, \quad (13)$$

where $\zeta_{rs} = (1 + \delta_{rs})^{1/2}/2$, and the matrix elements are related through

$$\omega_{rstu}^\gamma = \sum_{\Gamma} (-)^{s+t-\gamma-\Gamma} \left\{ \begin{matrix} r & s & \Gamma \\ u & t & \gamma \end{matrix} \right\} W_{rstu}^\Gamma[\Gamma], \quad (14)$$

$$W_{rstu}^\Gamma = \sum_{\gamma} (-)^{s+t-\gamma-\Gamma} \left\{ \begin{matrix} r & s & \Gamma \\ u & t & \gamma \end{matrix} \right\} \omega_{rstu}^\gamma[\gamma]. \quad (15)$$

Replacing pairs by single indices $rs \equiv x$, $tu \equiv y$ in eq. (12) and $rt \equiv a$, $su \equiv b$ in eq. (13), we bring the matrices W_{xy}^Γ and $f_{ab}^\gamma \equiv f_{rstu} = \zeta_{rs}\zeta_{tu}$, to diagonal form through unitary transformations $U_{xk}^\Gamma, u_{ak}^\gamma$:

$$U^{-1} W U = E \implies W_{xy}^\Gamma = \sum_k U_{xk}^\Gamma U_{yk}^\Gamma E_k^\Gamma \quad (16)$$

$$u^{-1} f u = e \implies f_{ab}^\gamma = \sum_k u_{ak}^\gamma u_{bk}^\gamma e_k^\gamma, \quad (17)$$

and then,

$$H_M = \sum_{k, \Gamma} E_k^\Gamma \sum_x U_{xk}^\Gamma Z_{x\Gamma}^+ \cdot \sum_y U_{yk}^\Gamma Z_{y\Gamma}, \quad (18)$$

$$H_M = \sum_{k, \gamma} e_k^\gamma \left(\sum_a u_{ak}^\gamma S_a^\gamma \sum_b u_{bk}^\gamma S_b^\gamma \right)^0 [\gamma]^{1/2}, \quad (19)$$

which we call the E and e representations. Since \mathcal{H}_m contains all the $\gamma = 00$ and 01 terms, for \mathcal{H}_M , $\omega_{rstu}^{00} = \omega_{rstu}^{01} = 0$. There are no one-body contractions in the e representation because they are all proportional to $\omega_{rstu}^{0\tau}$.

The eigensolutions in eqs. (18) and (19) using the KLS interaction [10,11]—in ref. [3] it is explained in detail why this venerable choice is a good one—for spaces of one and two major oscillator shells. The density of eigenvalues (their number in a given interval) in the E representation is shown in fig. 2 for a typical two-shell case.

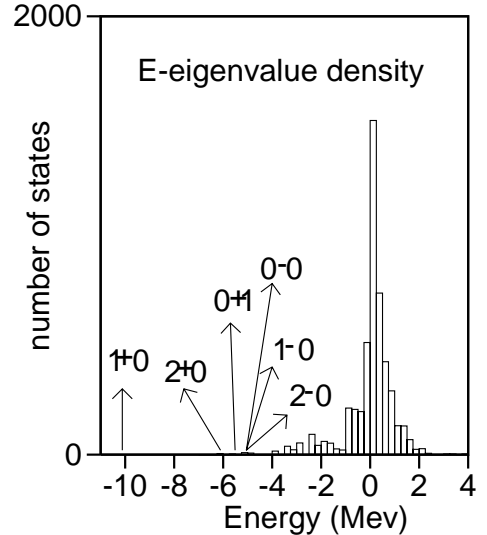


FIG. 2. E -eigenvalue density for the KLS interaction in the pf+sdg major shells $\hbar\omega = 9$. Each eigenvalue has multiplicity $[\Gamma]$. The largest ones are shown by arrows.

It is skewed, with a tail at negative energies which is what we expect from an attractive interaction.

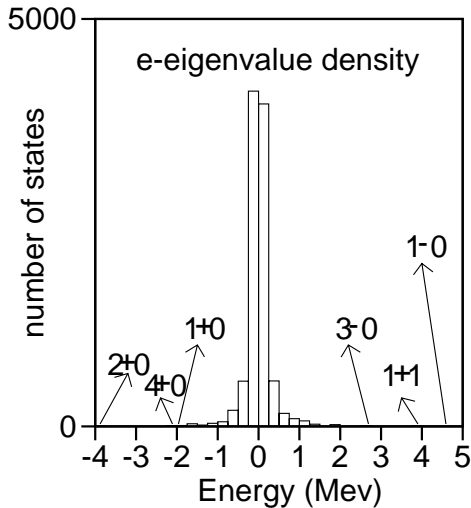


FIG. 3. e -eigenvalue density for the KLS interaction in the pf+sdg major shells. Each eigenvalue has multiplicity $[\gamma]$. The largest ones are shown by arrows.

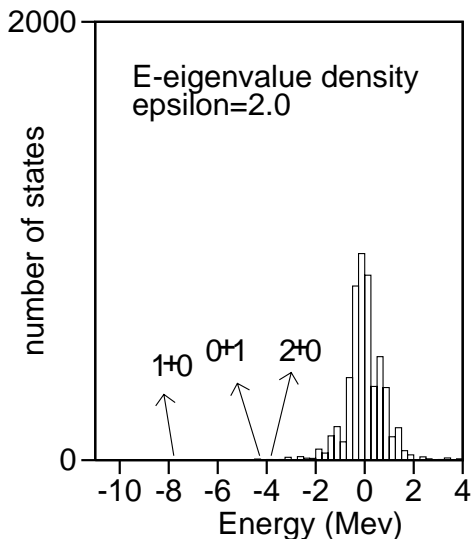


FIG. 4. E -eigenvalue density for the KLS interaction in the pf+sdg major shells $\hbar\omega = 9$, after removal of the five largest multipole contributions. Each eigenvalue has multiplicity $[\Gamma]$. The largest ones are shown by arrows.

The e eigenvalues have a number of simple properties demonstrated in [3, Appendix B]: their mean value always vanishes, their width is $\sqrt{1/8}$ of that of the E distribution, and they are twice as numerous. In

fig. 3 we find that they are very symmetrically distributed around a narrow central group, but few of them are neatly detached. The strongest have $\gamma^\pi = 1^-0, 1^+1, 2^+0, 3^-0, 4^+0$. **If the corresponding eigenvectors are eliminated from H in eq. (19) and the associated H in eq. (18) is recalculated, the E distribution becomes quite symmetric.** (The boldface is meant to call attention to the one very bad erratum in [3]). The result is shown in Fig. 4. The residual skewness is entirely accounted for by the $\Gamma = 1^+0, 0^+1$ and 2^+0 peaks, whose strength is somewhat eroded but remains substantial.

This result is most telling because from the work of Mon and French [12] (see also the references quoted in [1]) we know that a symmetric E distribution will lead to spectra in the m -particle systems that are identical to those of a random matrix. Therefore, we have found that - with the exception of three Γ peaks - the very few dominant terms in the e -distribution are responsible for deviations from random behaviour in H_M . Positively stated, these terms are at the origin of collective properties.

If the diagonalizations are restricted to one major shell, negative parity peaks are absent, but for the positive parity ones the results are practically identical to those of Figs. 2 and 3, except that the energies are halved. This point is crucial:

If u_{p_1} and u_{p_2} are the eigenvectors obtained in shells p_1 and p_2 , their eigenvalues are approximately equal $e_{p_1} \approx e_{p_2} = e$.

When diagonalizing in $p_1 + p_2$, the unnormalized eigenvector turns out to be $u_{p_1} + u_{p_2}$ with eigenvalue e .

In the figures the eigenvalues for the two shell case are doubled, because they are associated with normalized eigenvectors.

To make a long story short: The contribution to H_M associated to the $\Gamma = 01$, and $\gamma = 20$,

$$H_{\bar{P}} = -\frac{\hbar\omega}{\hbar\omega_0} |E^{01}| (\bar{P}_p^+ + \bar{P}_{p+1}^+) \cdot (\bar{P}_p + \bar{P}_{p+1}) \quad (20)$$

$$H_{\bar{q}} = -\frac{\hbar\omega}{\hbar\omega_0} |e^{20}| (\bar{q}_p + \bar{q}_{p+1}) \cdot (\bar{q}_p + \bar{q}_{p+1}), \quad (21)$$

turn out to be (naturally) the usual pairing plus quadrupole Hamiltonians, *except that the operators for each major shell of principal quantum number p are affected by a normalization*. E^{01} and e^{20} are the one shell values called generically e in the discussion above. To be precise

$$\bar{P}_p^+ = \sum_{r \in p} Z_{rr01}^+ \Omega_r^{1/2} / \Omega_p^{1/2}, \quad (22)$$

$$\bar{q}_p = \sum_{rs \in p} S_{rs}^{20} q_{rs} / \mathcal{N}_p, \quad (23)$$

where

$$\Omega_r = j_r + 1/2, \quad q_{rs} = \sqrt{\frac{1}{5}} \langle \| r^2 Y^2 \| s \rangle,$$

$$\Omega_p = \sum_r \Omega_r = \frac{1}{2} D_p \quad \mathcal{N}_p^2 = \Sigma q_{rs}^2 \cong \frac{5}{32\pi} (p + 3/2)^4, \quad (24)$$

More on the other collective terms of H_M in section III

A. Collapse avoided

The pairing plus quadrupole model has a long and glorious history [13,14], and one big problem: as more shells are added to a space, the energy grows, eventually leading to collapse. The only solution is to stay within limited spaces, but then the coupling constants have to be readjusted on a case by case basis. The normalized versions of the operators presented above are affected by universal coupling constants that do change with the number of shells. Knowing that $\hbar\omega_0 = 9$ MeV, they are $|E^{01}|/\hbar\omega_0 = g' = 0.32$ and $|e^{20}|/\hbar\omega_0 = \kappa' = 0.216$ in Eqs. 20 and 21. (One should not forget that for practical use, these numbers must be renormalized. See [3] for details.)

Introducing $A_{mf} \approx \frac{2}{3}(p_f + 3/2)^3$, the total number of particles at the middle of the Fermi shell p_f , the relationship between g' , κ' , and their conventional counterparts [13] is, for one shell

$$\frac{0.32\hbar\omega}{\Omega_p} \cong \frac{19.51}{A^{1/3}A_{mf}^{2/3}} = G \equiv G_0 A^{-1},$$

$$\frac{0.216\hbar\omega}{\mathcal{N}_p^2} \cong \frac{1}{2} \frac{216}{A^{1/3}A_{mf}^{4/3}} = \frac{\chi'}{2} \equiv \frac{\chi'_0}{2} A^{-5/3}. \quad (25)$$

What I propose next, is to show that both variants do indeed produce shell effects in $A^{1/3}$, a necessity emphasized at the beginning of section IA 1. Then, by allowing the particles to be promoted to shells well above the Fermi one, it will become clear why the conventional forms produce collapse, and the new ones do not.

Assume $m = O(D_f)$ particles in shell p_f , for which, $D_f = O(A^{2/3})$.

Consider first the pairing force. On a space of degeneracy D , it produces an energy

$$E_P = -\frac{|G|}{4} m(D - m + 2) = -|G|O(mD).$$

The first equality is a standard result. For $D = D_f$, the conventional choice $G = O(A^{-1})$ leads to $E_P = O(A^{1/3})$, i. e., it guarantees the correct scaling for shell effects. That the result is not as trivial as it looks can be gathered from this quotation concerning the G coupling: “We know of no reliable way of predicting this A^{-1} dependence...” [13].

For a quadrupole force, an estimate for the energy can be obtained by constructing a determinantal state that maximizes the quadrupole moment

$Q_0 = \sum_{i=1}^n (2n_{zi} - n_{xi} - n_{yi})$, where n_{xi}, n_{yi}, n_{zi} are the number of quanta. The largest term in the sum is then $2p$, the next $2p - 3$, then $2p - 6$, etc. Therefore $Q_0 = O(mp)$, and

$$E_q \approx -|\chi'|Q_0^2 = -|\chi'|O(m^2D),$$

which in turn explains (for $D = D_f$) the origin of the usual choice $\chi' = O(A^{-5/3})$ for the quadrupole strength, that leads to $E_q = O(A^{1/3})$.

It is clear from equations (25) that the operators are affected by coefficients that go as $A^{-1/3}D^{-1}$ (instead of A^{-1}) for pairing, and as $A^{-1/3}D^{-2}$ (instead of $A^{-5/3}$) for quadrupole. For $D = D_f$, the energies are again $O(A^{1/3})$, but now this important empirical fact is a direct consequence of the interaction. For arbitrary D , the energies of the traditional (old) versions transform into the normalized (new) one as

$$E_P(\text{old}) = O\left(\frac{mD}{A}\right) \implies E_P(\text{new}) = O\left(\frac{m}{A^{1/3}}\right),$$

$$E_q(\text{old}) = O\left(\frac{m^2D}{A^{5/3}}\right) \implies E_q(\text{new}) = O\left(\frac{m^2}{A^{1/3}D}\right).$$

If the m particles are promoted to some higher shell with $p = p_f + M$, $D \approx (p_f + M)^2$, both energies grow in the old version. For sufficiently large M - because of the term in M^2 - the gain will become larger than the monopole loss $O(mM\hbar\omega) = O(MA^{1/3})$ that is only linear in M . Therefore the traditional forces lead the system to collapse. In the new form there is no collapse: E_P stays constant, E_q decreases and the monopole term provides the restoring force that guarantees that particles will remain predominantly in the Fermi shell.

Let us go back now to the monopole term W that plays such a crucial role in \mathcal{H}_m^d . The collective operators are obviously m_p , and the “conventional” monopole force $V_0mm = V_0(\sum m_p)^2$. Granted that V_0 would be scaled by A^{-1} , as a centroid, it would have to be calculated for the occupied orbits only, which would have to be varied to detect a minimum. In other words: there is no collapse, but the operator is useless: All the information is contained in V_0 , which is not a constant, but the result of HF variation from nucleus to nucleus.

However, it is clear that W in Eq. (10) is the normalized version of the mm operator, and that it is quite a useful.

III. PAIRING MISCELLANEA

The treatment of the monopole term is new, and I can only hope that the techniques and results in [2] will stimulate further interest in \mathcal{H}_m .

The quadrupole force is very old, and nobody seems to have much problems concerning it. Here I only note that

the quadrupole terms involving $2\hbar\omega$ jumps are quite suppressed in the realistic force: They happen to be exactly what is needed to produce the correct effective charges in perturbation theory. Therefore we are left basically with a $q \cdot q$ force of Elliott's type, associated with the SU(3) symmetry [15].

The other big multipole terms in the collective Hamiltonian are very much what they should be: octupole, hexadecapole and $\sigma\tau$ forces. There is also a huge $\gamma = 10^-$ center of mass contribution, that is nicely decoupled, as it should, to ensure momentum conservation.

We are left with the pairing terms. Contrary to quadrupole—which seems to be naturally and satisfactorily included in mean field formulations—pairing has to be put by hand. Since the old version is not sufficient for state of the art calculations, there are many proposals to replace it. Furthermore, there is also much interest in the other pairing terms. Let us start with a look at these.

The very large $\Gamma = 10$ one is the $ST = 10$ part of a pairing force in LS scheme ($\Gamma = 01$ is indeed the $ST = 01$ part). In jj coupling, the term is massively dominated by the matrix elements between the largest $l \cdot s$ partners (e. g., $f_{7/2}f_{5/2}$ in $p = 3$). In spite of its strength it is much suppressed by the splitting between the orbits. Furthermore, its $T = 0$ nature makes it rapidly inefficient when moving away of $T = 0$ nuclei. It is also at the origin of a common misconception regarding the Wigner term, that deserves a digression.

The Wigner term is the piece of the force that goes like T . It simply comes from the $T(T + 1)$ symmetry energy which is very strong, and produces a cusp in the binding energies at $T = 0$. Therefore, before deciding that $\Gamma = 10$ (or some $J0$) pairing is responsible for the Wigner term, people are asked to check that the coefficient of the symmetry energy (a centroid of type b in Eq. (6)) has been kept constant.

The $\Gamma = 20$ term is a puzzle. I do not know what to make of it.

There are no other pairing terms that may claim a collective status, in particular, the $\Gamma = 21$ term does not seem to amount to much. I would strongly urge people to stop using it.

Finally, let me return to $\Gamma = 01$ pairing. As mentioned, there are various new candidates. Once the ones that do not make sense are discarded (I have in mind the δ force), my feeling is that something like the version that has been proposed here will be either accepted or independently discovered. Modifications may be necessary, because there are problems (discussed in [3]), but the proposal seems quite sound.

Refs. [16–18] contain discussions about the influence of $\Gamma = 01, 10$ pairing on backbending rotors, that can be summed up as follows:

Although the energetics of the yrast band are strongly affected by the pairing modifications, the other properties are not, since the wavefunctions change little.

Nuclear physics is not confined to backbending rotors. In other regions, pairing will dominate the coupling schemes, with quadrupole acting as a perturbation. In general, it seems quite likely that nuclear structure will remain a field in which pairing and quadrupole are the main players, with monopole acting as referee.

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- [1] At the conference I was originally supposed to talk about pairing, and I tried to convey the main ideas in a few minutes. For an expanded version of the main presentation, which dealt with level densities see <http://xxx.LANL.gov/abs/nuc-th/9910002,3>
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